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Temperature and enviromental effects on the fatigue fracture in polystyrene

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Fatigue tests have been conducted on polystyrene in air at different mean stress conditions and also over the temperature range − 60 to 60° C using notched specimens. In addition, some tests were performed with the specimens immersed in a detergent and in a corn oil. The data were first represented on a conventional log ΔK–log da/dN plot to determine the Paris law parameters and how these varied with mean stress, temperature and environment. Considerable variations were noted but no useful pattern could be discerned. The data were then analysed using a recently proposed two-stage line plastic zone model so that a crack tip zone stress and a fatigue damage factor could be found. These stresses were found to be very high in air and to increase with increasing mean stress, and this is attributed to a high degree of constraint at the crack tip. There was an abrupt change in σ_c at about 20° C which, it is suggested, is caused by the onset of crazing due to the presence of a loss peak. Associated changes in K_c were also noted. In environments, the stresses were much smaller and agreed with those obtained in static fracture, suggesting that crazes are formed in environments prior to, and not during, fatigue crack growth. The stress reduction factor, α, remained constant at about 0.2 with both temperature and environment.

1. Introduction

The analysis of experimental data for fatigue crack propagation in polymers has mainly been based on the Paris power-law equation [1–4]. To account for frequency and mean stress effects, additional empirical parameters have been incorporated into the Paris equation to fit the data. Although this approach is reasonably successful for the description of fatigue crack growth in polymeric materials, it lacks a theoretical basis and any link with established fracture parameters, such as crack-opening displacement and crack stress. Recently, however, Williams [5], using the Dugdale line plastic zone model and the assumptions of a critical crack-opening displacement[†] and a constant cyclically reduced craze stress, presented a model of fatigue crack growth in polymers. This has been shown to give a good description of fatigue crack-growth rate, mean stress and frequency effects for PMMA, Nylon 66 and polycarbonate [5]. In view of the success of this new model, it has been employed in this investigation for the analysis of fatigue crack growth in a commerical grade polystyrene (PS).

There has been a considerable amount of work on crazing and fracture of polystyrene under monotonic increasing load condition [6–10]. However, few investigations on the fatigue fracture in polystyrene using pre-cracked or notched specimens are reported. Most previous studies [11, 12] on fatigue are performed on unnotched specimens and this makes the interpretation of the

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[†] The critical opening displacements for fatigue and static fracture are not necessarily identical.

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experimental results difficult in fracture mechanics terms. Skibo *et al.* [13] and Manson and Hertzberg [14] seem to be the first investigators to report fatigue fracture in PS using fracture mechanics techniques. Their studies, however, were concerned with the frequency effect on fatigue crack growth, and no other factors which affect the crack growth rate were investigated. The purpose of this paper is, therefore, to describe the effects of some of these other factors, such as mean stress, temperature and liquid environment, in terms of both the Paris law and the Williams model.

2. Experimental procedure

The commercial grade polystyrene used in this investigation was obtained* in the form of 3 mm thick sheets. Single-edge notched (SEN) specimens with dimensions 150 mm × 50 mm were prepared with machined notches of initial crack length approximately 9 mm. The stress-intensity amplitude (ΔK) is given by:

$$\Delta K = \Delta\sigma\sqrt{a}\,Y$$

where $\Delta\sigma$ is the stress amplitude, a is the crack length, and Y is the geometric correction factor obtainable from the boundary collocation results of Brown and Strawley [15].

All fatigue experiments were performed in an Instron testing machine equipped with a temperature box which could be controlled to an accuracy of ± 1° C by a "Eurotherm" unit. The actual temperature of the specimen was measured by a thermocouple fixed near the crack tip. A test frequency of 0.15 Hz was chosen and the load wave-form of the Instron machine was approximately triangular. Crack growth rates were measured using a travelling microscope to an accuracy of 0.01 mm. For the environmental fatigue tests, glass plates and silicon rubber were used to construct small reservoirs around the cracked section of the specimen to contain the liquid.

The following four series of fatigue experiments were then performed:

(a) temperature varying from − 60 to 60° C in a laboratory air environment;

(b) temperature at 30° C and stress intensity ratio (R) varying from 0 to 0.6;

(c) a detergent (Adinol) environment with temperature at 40 and 60° C, respectively; and

(d) a corn oil (Mazola) environment with temperatures between 20 and 60° C.

Each set of the experimental fatigue crack-growth data presented in Figs. 1, 2, 4 and 5 are obtained from one specimen only. Repeat tests showed, however, that there was little difference between nominally identical specimens tested under similar conditions, but rate-specimen variability could still occur in the results.

3. Results and discussion

3.1. Temperature, stress-intensity ratio and environmental effects

The effect of temperature (T) on fatigue crack-growth rate (da/dN) of polystyrene in a laboratory air environment and at 0.15 Hz is demonstrated in Fig. 1 plotted on the conventional log–log basis. The temperature range is from − 60 to 60° C and the stress intensity ratio $R = 0$. In broad terms, for a given ΔK, crack growth rate increases with increasing temperature. For example, when $\Delta K = 1.0\,\text{MPa}\sqrt{\text{m}}$, $da/dN = 0.5$, 1.2, 2.4, 6.0 and 10 μm/cycle for $T = -20$, 0, 20, 40 and 60° C, respectively. The sigmoidal

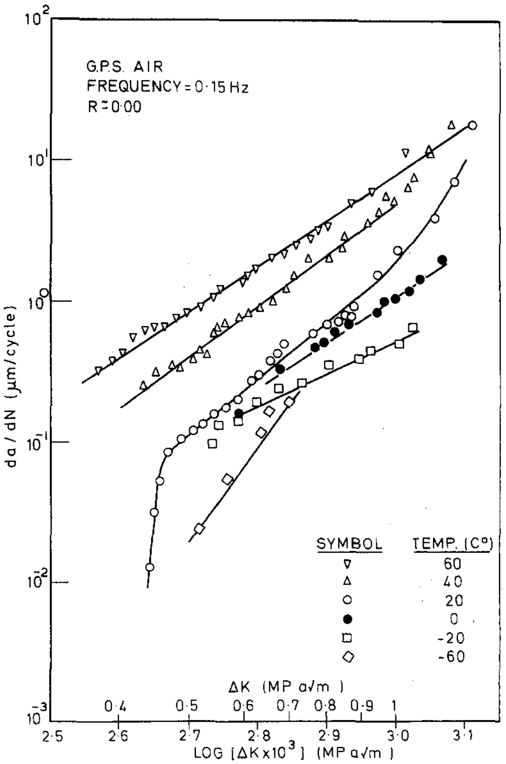

Figure 1 Fatigue crack-growth rate for PS at varying temperatures (log–log).

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*Kindly supplied by BP Chemicals Limited.

TABLE I Values of A and m in the Paris equation* at different temperatures

T(° C)	$\log_{10}A$	m	$\mathrm{d}a/\mathrm{d}N$ (μm/cycle)
60	0.91	3.1	0.3 to 20
40	0.76	3.7	0.2 to 20
30	0.74	3.8	0.1 to 10
20	0.39	4.3	0.1 to 7
0	0.07	3.6	0.1 to 2
−20	−0.26	2.6	0.1 to 1
−60	0.48	7.4	0.01 to 0.2

*$\mathrm{d}a/\mathrm{d}N = A\ (\Delta K)^m$, where $\mathrm{d}a/\mathrm{d}N$ is in μm/cycle and ΔK in MPa $\sqrt{\mathrm{m}}$.

shape of the $\mathrm{d}a/\mathrm{d}N$ versus ΔK curve is displayed by the 20° C data. $\mathrm{d}a/\mathrm{d}N$ values below −60° C are very small and have not been obtained in this investigation. It seems that the experimental data for −20, 0 and 20° C converge at $\Delta K \simeq 0.5\ \mathrm{MPa}\sqrt{\mathrm{m}}$, but it is likely that each temperature curve will have its own threshold ΔK value.

Table I shows the values of A and m in the Paris equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A(\Delta K)^m \qquad (1)$$

at different temperatures, taken from Fig. 1, and the range of $\mathrm{d}a/\mathrm{d}N$ values used is also indicated. There does not seem to be a trend for m with temperature variation, although high m values result from low growth data, as at −60° C.

Fig. 2 shows the mean stress effect on fatigue crack growth by varying the stress intensity ratio, R. Since these experiments were conducted on SEN specimens with tension–tension cyclic stresses, the mean stress-intensity level (K_m) was not constant but was proportional to incremental ΔK values, i.e. at a given ΔK, as R increased, K_m also increased according to the following relations:

$$K_\mathrm{m} = \frac{\Delta K}{2}\left(\frac{1+R}{1-R}\right), \qquad (2)$$

where

$$R = \frac{K_\mathrm{min}}{K_\mathrm{max}} = \frac{\sigma_\mathrm{min}}{\sigma_\mathrm{max}}.$$

Thus, to see if any mean stress effect exists, it is only necessary to compare $\mathrm{d}a/\mathrm{d}N$ values at the same ΔK but varying R. The results shown in Fig. 2 suggest that the mean stress effect is relatively small in PS, although there is a slight $\mathrm{d}a/\mathrm{d}N$ increase (two- to four-fold) with increasing R.

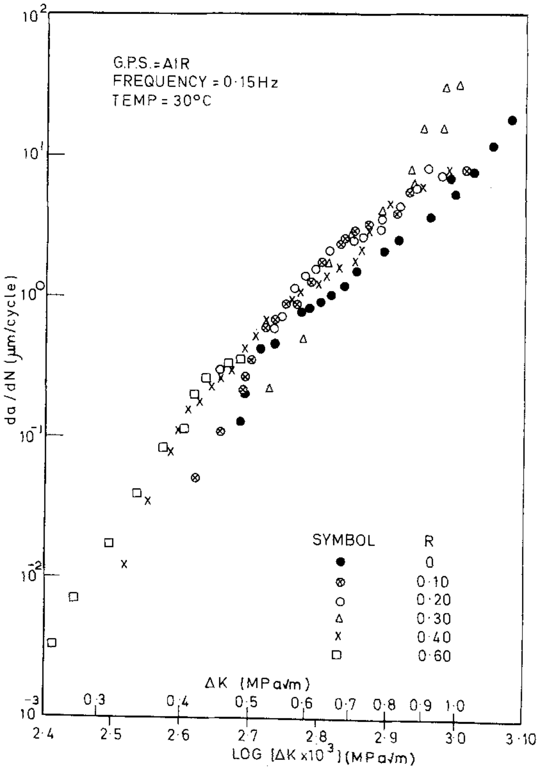


Figure 2 Effect of stress-intensity ratio (R) on fatigue crack-growth rate in PS at 30° C (log–log).

In this respect, PS differs from other visco-elastic polymeric materials, such as PMMA [1] and epoxy resins [16], where a much larger (typically an order of magnitude difference in $\mathrm{d}a/\mathrm{d}N$) mean stress effect exists. It is also noted that m in Equation 1 increased from 3.83 at $R = 0$ to 4.88 at $R = 0.04$.

Arad *et al.* [1] and Sutton [16] suggest that the mean stress effect on fatigue crack-growth data in some polymers can be rationalized by using either a parameter λ ($= K_\mathrm{max}^2 - K_\mathrm{min}^2$) or the cycle strain-energy release rate, ΔG, which are equivalent since $\Delta G = \lambda/E$. They have successfully shown that, at least for PMMA and an epoxy polymer, the fatigue data obey the following fatigue crack-propagation laws:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = B(\lambda)^n \qquad (3)$$

and

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta G)^p, \qquad (4)$$

where B, C, n and p are constants. A straight line is obtained in the log $\mathrm{d}a/\mathrm{d}N$ versus log λ or log

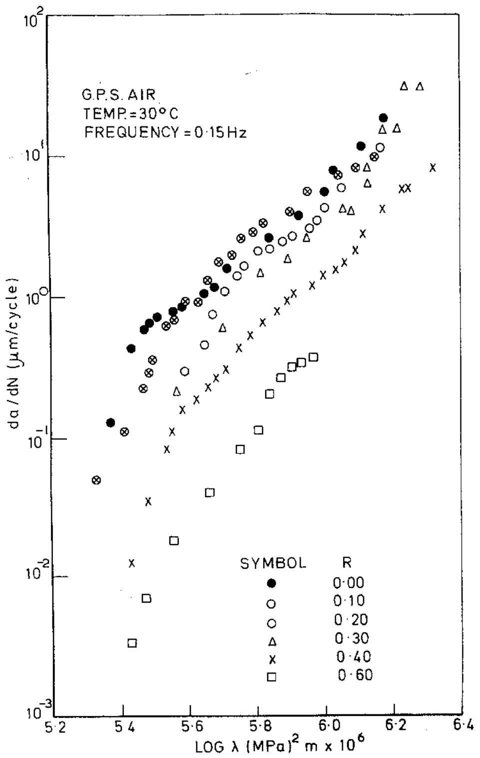
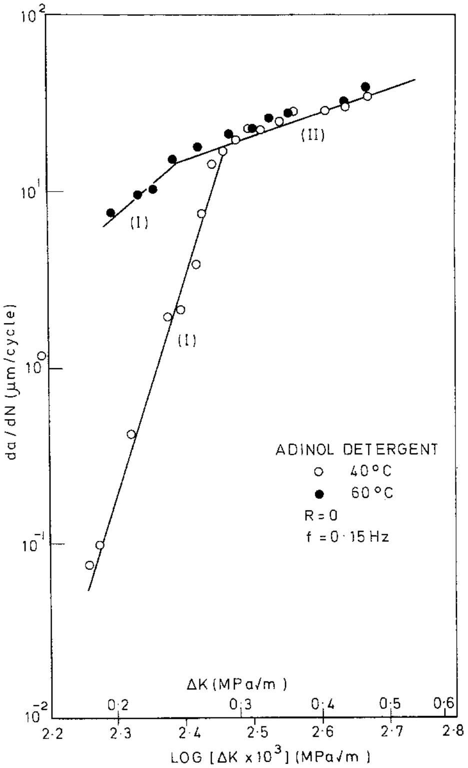


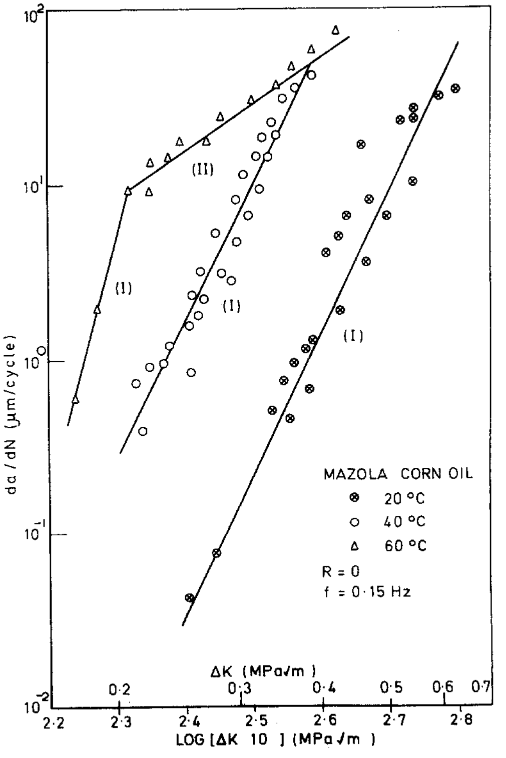
Figure 3 Fatigue crack-growth rate in PS as a function of λ (log–log).

ΔG plots, irrespective of the values of R. Although generality is claimed of Equation 3 by Radon *et al.* [17] in that $\mathrm{d}a/\mathrm{d}N$ can be determined from a single set of data, irrespective of the values of ΔK and R, this is obviously not the case for the present PS fatigue data, as shown in Fig. 3. For a constant λ, $\mathrm{d}a/\mathrm{d}N$ decreases markedly with increasing R, since ΔK decreases according to the equation:

$$\Delta K = \sqrt{\left[\left(\frac{1-R}{1+R}\right)\lambda\right]}. \tag{5}$$

Figs. 4 and 5 show the effect of an Adinol detergent and a Mazola corn oil, respectively, on fatigue crack growth in PS on a log–log basis. The simultaneous effects of temperature and liquid environment are clear when these two figures are compared with Fig. 1. For all the temperatures investigated, and at comparable ΔK values, $\mathrm{d}a/\mathrm{d}N$ is more than two orders of magnitude higher in the liquid environments than in air. This effect is somewhat more pronounced in corn oil than in detergent and at higher rather than lower temperatures. It may also be noted that, in broad terms, the environmental fatigue data can be divided into two regions (particularly for $T = 40$ and 60° C), in which





Figures 4 and 5 Fatigue crack-growth rate for PS in detergent (Adinol) and corn oil (Mazola) (log–log).

the lower da/dN range (region I) has a higher m value and the higher da/dN range (region II) has a lower m magnitude ($m \simeq 2$). From the trend of these fatigue data (Fig. 4 and 5), it could be suggested that region II is unique for all temperatures for a given liquid environment, except that the slope transition ΔK values will increase with decreasing temperature and occur at higher da/dN magnitudes. This aspect, however, has not been studied in detail in this work and more data are needed to substantiate this proposal. The values of A and m in the Paris equation for these two regions and in these two liquid environments are given in Table II, and may be contrasted with those A and m values shown in Table I.

3.2. Theoretical analysis of fatigue crack-growth data

Williams [5] has recently attempted to model fatigue crack-propagation behaviour in polymers from the continuum mechanics of the crack tip based on Dugdale's line plastic zone analysis. He assumed that the cyclic effect was to reduce the craze stress and that the reduction was dependent on the degree of unloading. Repeated cycling, therefore, resulted in the growth of the craze with a corresponding increase of the crack-opening displacement (COD). When the COD reached a certain critical value, crack growth occurred* and the rate of crack growth, da/dN, could be related to the applied stress intensity factor, K, and the critical static stress intensity factor, K_c, by:

$$\frac{da}{dN} = \frac{\pi}{8(1-\alpha)^2}\frac{1}{\sigma_c^2}[K^2 - \alpha K_c^2] \qquad (6)$$

TABLE II Values of A and m in Paris equation* at different temperatures in detergent and in corn oil

Environment	T(° C)	$\log_{10}A$	m	Region
Adinol detergent	60	2.4	1.9	II
	40	7.8	12.2	I
	40	2.4	1.9	II
Mazola corn oil	60	2.8	2.7	II
	60	10.7	14.3	I
	40	4.8	7.7	I
	20	3.4	8.1	I

*$da/dN = A(\Delta K)^m$, where da/dN is in μm/cycle and ΔK in MPa$\sqrt{m}$.

where σ_c is the crack tip stress which reduces to $\alpha\sigma_c$ on fatiguing, and:

$$\alpha = (1-f) + fR^2 \qquad (7)$$

where f is the damage factor.

This model will now be employed to analyse the fatigue data of PS described in the previous section in the hope of elucidating the mechanics of the fatigue process. Fig. 6 shows the fatigue data obtained at 30° C and $R = 0$ to 0.6 according to Equation 6. The predicted linear relationships are obtained, but there are abrupt transitions at high K and R values. By plotting the intercept K^2 values at $da/dN = 0$ for these curves as a function of R^2 (Fig. 7), a straight line is obtained which agrees with Equation 7. By extrapolation at $R^2 = 1$ and $\alpha = 1$, we obtain $K_{IC} = 1.02$ MPa$\sqrt{m}$ and $f = 0.76$. This K_{IC} value agrees very well with that obtained by Marshall *et al.* [6]. If, in addition, the intercept K^2 values for the portions of the curves above the transitions are plotted against R^2, there is considerable scatter, giving a very approximate value of $K_{IC} \simeq 1.90$ MPa$\sqrt{m}$. This high K_{IC} value could be a result of the fracture mode transition from a smooth, mirror-like surface (i.e. very sharp crack-tip radius) to a misty appearance due to multiple crazing on either side of the crack plane [13] (i.e. a blunted crack tip).

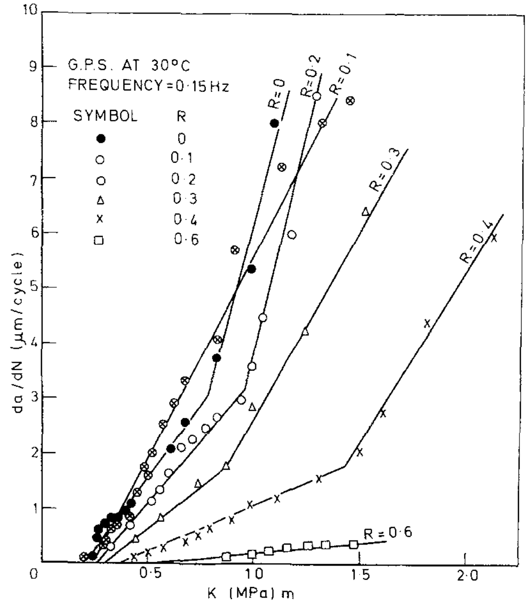


Figure 6 Fatigue crack-growth data for PS plotted in accordance with Equation 6 for varying R.

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*da/dN is equated to the craze growth per cycle here and assumes unsteady state. If the growth is discontinuous, as described in [13], then the model will apply, provided an average growth per cycle (in this case, the craze) is used.

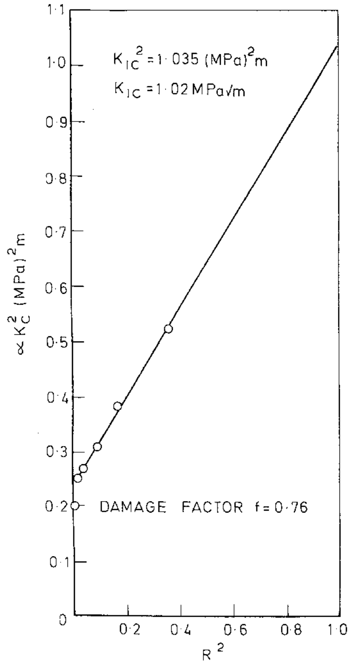


Figure 7 Intercept at $da/dN = 0$ as a function of R^2.

The craze stress can be calculated from the slopes of the lines in Fig. 6, and Equation 6, and the values are given in Table III. There is a rapid increase in σ_c with increasing R and the values are considerably in excess of static values. The fatigue craze stress, $\alpha\sigma_c$, is also given and this is similarly much higher than the static case. The most likely explanation of this effect is that the unfatigued zone is deeply embedded within the material and is subject to a high degree of constraint which increases the effective yield stress of the material. Higher values of α will increase this constraint, thus increasing σ_c. Similar results are obtained from the data in [5] for PMMA and Nylon 66. As further support for this argument, it should be noted that at the higher K values, after the transition in the da/dN versus K^2 graphs, σ_c remains fairly constant at about $300\,MN\,m^{-2}$, suggesting that, at larger deformations, the constraint does not increase and that the transition is from a high to a lower constraint condition.

TABLE III Derived values of αK_c^2, α, σ_c and $\alpha\sigma_c$ at 30° C with frequency = 0.15 Hz and $K_c^2 = 1.035\ (MPa)^2 m$. Stress ratio R varying

R	$\alpha K_c^2\ (MPa)^2 m$ at $da/dN = 0$	α	σ_c(MPa)	$\alpha\sigma_c$(MPa)
0–0.1	0.25	0.24	298	72
0.2	0.27	0.26	387	101
0.3	0.31	0.30	501	150
0.4	0.38	0.37	756	277
0.6	0.52	0.51	2015	1021

The temperature data are replotted in Fig. 8 and linearity is evident with slope transitions about 20° C. The αK_c^2 values obtained from $da/dN = 0$ are shown in Table IV and plotted as a function of temperature in Fig. 9. There is a very slow change with temperature below 20° C, but there is a sharp decrease thereafter. This type of behaviour is

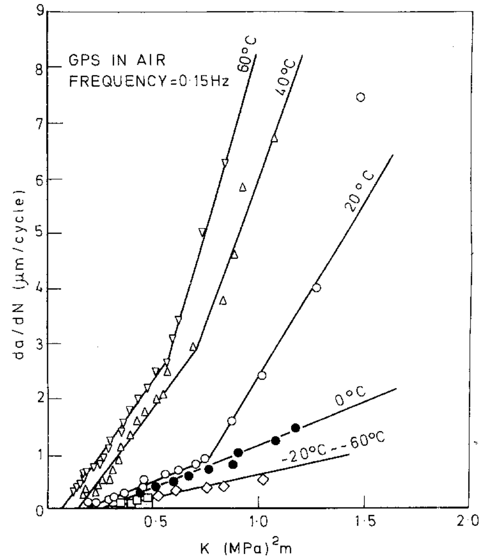


Figure 8 Fatigue crack-growth data for PS at varying temperatures plotted in accordance with Equation 6.

TABLE IV Derived values of $\alpha\ K_c^2$ and σ_c at varying temperatures ($R = 0$, frequency = 0.15 Hz) in air and environments

T(° C)	In air		In corn oil		In detergent	
	αK_c^2 (MPa)² m	σ_c (MPa)	αK_c^2 (MPa)² m	σ_c (MPa)	αK_c^2 (MPa)² m	σ_c (MPa)
60	0.071	337	0.026	35	0.022	37
40	0.15	336	0.062	46	0.058	30
30	0.25	283	–	–	–	–
20	0.19	620	0.106	85	–	–
0	0.26	622	–	–	–	–
−20 −60	0.29	861	–	–	–	–

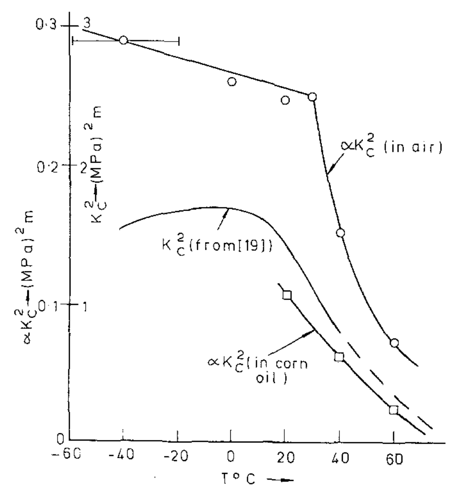


Figure 9 Intercept at $da/dN = 0$ as a function of temperature in both air and corn oil and K_c^2 in air as a function of temperature.

common in K_c measurements in polystyrene where the occurrence of the β transition around 20° C [18] results in craze formation and a corresponding decrease in K_c. Also shown are K_c^2 values taken from [19] which show a similar effect and indicate a value of $\alpha = 0.2$ to be approximately constant for the whole temperature range. The σ_c values determined from the slopes in Fig. 8 and using $\alpha = 0.2$ are shown in Fig. 10, together with $\alpha\sigma_c$ ($\alpha = 0.2$), and a very distinct change in σ_c at about 20° C is indicated. This corresponds to the point where K_c starts decreasing (Fig. 9) because of the crack tip craze formation, and this is reflected in a sharp drop in σ_c. The formation of the craze is most likely to be caused by the presence of the β peak at about this temperature which enables stable craze growth to occur. Large changes in fatigue rate at temperatures corresponding to β peaks previously reported [20] are, therefore, likely to be changes in deformation mode (i.e. the onset of crazing), resulting in an abrupt change in σ_c and a changing K_c.

Fig. 11 and 12 show the data obtained in liquids plotted in terms of the model. The results are rather similar for each liquid and this is apparent in Table IV, where the αK_c^2 values are given. Those for corn oil are shown in Fig. 9 and indicate a similar form to the air data, but at a reduced level. Separate static K_c measurements show a reduction of a factor of 0.7 and, if this is applied to the results, a constant value of $\alpha = 0.15$ describes the data. Using this value of α with the slopes in Fig. 11 and 12 gives the σ_c values shown in Table IV. These are much less than the air values and close to the expected craze stress values, confirming that the environments plasticize the crack-tip region and

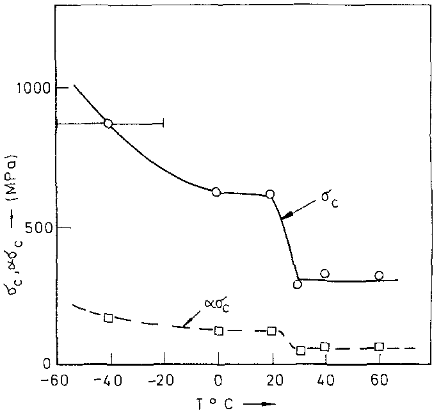
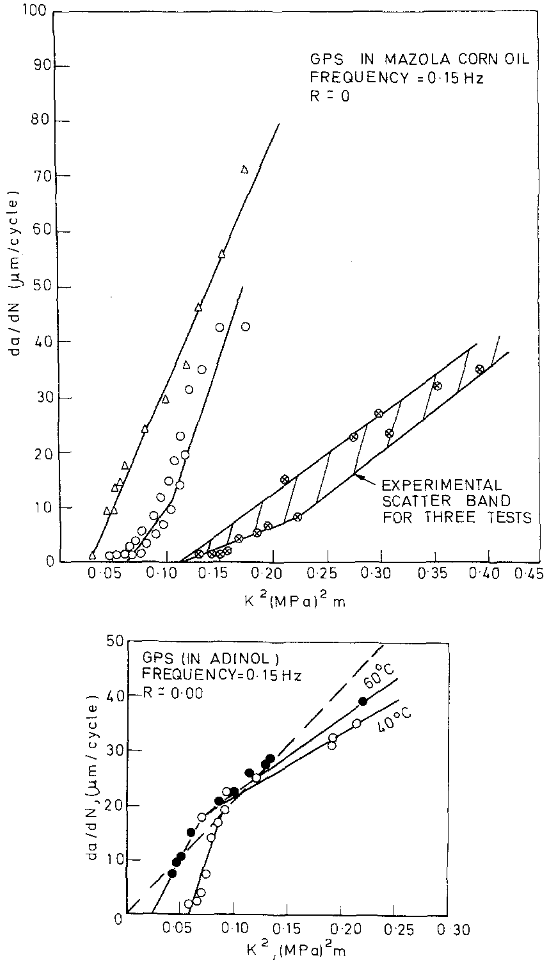


Figure 10 Crack-tip stresses in air as a function of temperature.



Figures 11 and 12 Fatigue crack-growth data for PS in detergent and corn oil plotted according to Equation 6.

generate an initial craze which is further reduced by fatigue. There is no evidence of any genuine unique stage-II curve when plotted on this basis, but the high growth data in Adinol and that at 60° C in corn oil could be represented approximately by the same line through the origin (the broken line in Fig. 12) which suggests that under these conditions $\alpha \rightarrow 0$, i.e. immediate fracture on fatiguing with $\sigma_c = 52\,\text{MPa}$, and that the data are really viscoelastic slow-crack growth. This does, of course, explain why $m \rightarrow 2$ in the Paris plots in this region.

4. Conclusions

It is clear that representing the data discussed here in terms of the Paris law gives wide variations in the parameters A and m, and it is difficult to assign physical significance to their values. Attempts to describe mean stress effects were similarly unsatisfactory. The use of the two-stage line plastic zone model does provide numbers, which, although problematical in terms of their magnitude in some cases, do provide a physical basis for the mechanisms involved. In PS, as in other polymers, it appears that with stress cycling at low K values a highly constrained zone is established at the crack tip where very high stresses occur. This is then reduced by the fatigue action by up to 80%, but is still substantially higher than static craze stresses. Fracture here occurs with very low crack-opening displacements of the order of 1 μm compared with 8 μm in static failure. It is perhaps not surprising that failure should occur at lower strains under high constraint conditions and this is confirmed when for higher mean stresses the craze stress increases and the displacement decreases. The energies to fracture undergo much less variation than either stresses or displacement but there are still substantial changes.

The effect of temperature is largely to change K_c but at 20° C, where there is a β transition, crazing occurs giving a sharp change in σ_c which is reflected in $\mathrm{d}a/\mathrm{d}N$. It seems likely that the presence of an active viscoelastic process is necessary to generate crazes which decrease σ_c. The effect is also apparent in K_c data where there is a steady change in the region of such a transition. Liquid environments reduce both K_c and σ_c as would be expected, and the σ_c values are close to those for static craze stresses. The crack-opening displacements are also close to the static value, confirming that the model of a fatigue-damaged craze is sound, but that in air a rather special situation exists in which very high constraints give high stresses. The proportion of damage by fatigue, α, seems to be quite constant with temperature and environment but does change with degree of constraint. As a final comment, it is worth noting that an apparent second stage of growth in environmental fatigue has been demonstrated, by the model, to be essentially immediate cracking at higher K values and serves to confirm the value of using such a representation.

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